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Excess thermal-noise in the electrical breakdown of random resistor networks

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Abstract. We discuss a new type of excess noise strongly sensitive to non-homogeneous Joule heating of random resistor network and associated with local sources of thermal noise. The evolution of the network towards an electrical breakdown of conductor-insulator type is then studied by using a biased percolation model and it is analysed in terms of an excess-noise temperature. Monte Carlo simulation results show a significant increase of the excess-noise temperature over the average temperature of the network. Remarkably the excess-noise temperature scales with the resistance with an exponent of about 3. The predictivity of the model can be tested on thin film resistors where the determination of the excess noise temperature should provide a valuable indicator of the defectiveness of the film.

PACS. 72.70.+m Noise processes and phenomena – 73.61.-r Electrical properties of specific thin films and layer structures (multilayers, superlattices, quantum wells, wires, and dots) – 81.70.Cv Nondestructive testing: ultrasonic testing, photoacoustic testing

1 Introduction

Excess noise is the noise contribution which algebraically adds to the thermal equilibrium value determined by Nyquist law. It is recognized as an interesting phenomenon both in fundamental and applied physics [1–16]. In the former case it is the microscopic source from which it originates which attracts the attention of scientists [1–4]. In the latter case, being a non-destructive indicator of device reliability and degradation, its measure represents an interesting parameter to define the quality of an electronic device [1–10].

Here we focus on the excess noise coupled to the resistance degradation process of a thin film and we simulate the electrical breakdown of the film by describing it as a two-dimensional random resistor network (RRN) of conductor-insulator type. The RRN indeed provides a general and powerful model for studying several properties of disordered systems and in particular it has been largely applied to discuss electrical conductivity and noise of thin film resistors [13–23]. In particular, by considering the film as an assembly of elemental regions of mesoscopic sizes, the present theoretical approach is particularly appropriate to investigate the regularity in the adhesion to the substrate and the homogeneity of films made of nanomaterials. We remark that dishomogeneities in the current distribution, due to the presence of defective regions inside a sample, can increase strongly the intensity of the electrical noise [6–9] well above the concomitant change of resistance.

While most of the attention in this context has been devoted to the $1/f$ type of excess noise [1–16], in this paper we introduce and investigate the properties of an excess thermal noise strongly sensitive to non-homogeneous Joule heating of a RRN. We analyze the properties of this new type of excess noise during the process of an irreversible change of the RRN resistance, the so-called electrical breakdown [15,17]. More precisely we study a degradation process associated with a systematic increase of the resistance [5–9,12–22]. The degradation is described by making use of a biased percolation model [17,22] and it is analysed in terms of the evolution of the excess-noise temperature. Monte Carlo simulations have been then performed to calculate this quantity. We found that the excess-noise temperature scales with the resistance with an exponent of about 3. Furthermore, the increase of the noise temperature with respect to both the average temperature of the RRN and the substrate temperature, associated with the increasing disorder of the network, can be considered as a sensitive indicator of the film defectiveness.

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2 Theoretical model and results

We describe a thin film as a two-dimensional square-lattice network, initially made of identical resistors and deposited on an insulating substrate at temperature T_0. Without losing in generality, we take a square geometry where N determines the linear sizes of the lattice and $N_{\mathrm{tot}} = 2N^2$ is the total number of resistors. In practical applications, the value of N can be related to the ratio between the size of the sample and the grain sizes. The network is contacted at the two opposite sides and a constant external current I is applied. We consider a degradation mechanism associated with an increase of the network resistance as a consequence of the generation of insulating defects (resistors with very high resistance *i.e.* broken resistors) [16,17,22]. The case of degradation mechanism associated with a resistance decrease and thus related to the presence of short-cutting defects (resistors with very low resistance) can also be studied [17,23].

Within the biased percolation model [22,23], we simulated the degradation process by using the Monte Carlo method and by taking an activated-energy expression for the probability of generating defects at the network position indexed by α:

$$W_\alpha = \exp(-\frac{E_o}{k_{\mathrm{B}} T_\alpha}) \tag{1}$$

where E_o is the activation energy characterizing the defect, k_{B} is the Boltzmann constant and T_α is the temperature of the α-th resistor. Indeed we assume that the current i_α flowing through each resistor r_α, is responsible for an extra Joule heating which implies an increase, δT_α, of the local temperature T_α. More precisely, we take [22]:

$$T_\alpha = T_0 + \delta T_\alpha = T_0 + A r_\alpha i_\alpha^2 \tag{2}$$

where A, measured in (kelvin/watt), is a parameter describing the temperature increase of each resistor coupled to the substrate which acts as a thermal reservoir at temperature T_0, due to heat from the Ohmic resistors. We neglect thus time-dependent effects associated with heat diffusion [21]. Then, starting from the perfect lattice, at each iteration step defects are generated according to W_α, consequently all local currents, temperatures and the associated probabilities are recalculated.

From the virtual-power theorem (Tellegen's theorem) [16,24], the spectral density of voltage fluctuations $S_V(f)$ of a two-terminal network under voltage operation mode can be written as:

$$S_V(f) = \sum_\alpha s_{v\alpha}(f) \left(\frac{i_\alpha}{I}\right)^2 \tag{3}$$

where $s_{v\alpha}(f)$ is the voltage spectral density of the branch α and the sum is extended over the N_{tot} network branches. For the excess thermal-noise we consider the white region of the spectrum and, in analogy with the Nyquist formula, we take:

$$s_{v\alpha} = 4\ k_{\mathrm{B}}\ T_\alpha\ r_\alpha. \tag{4}$$

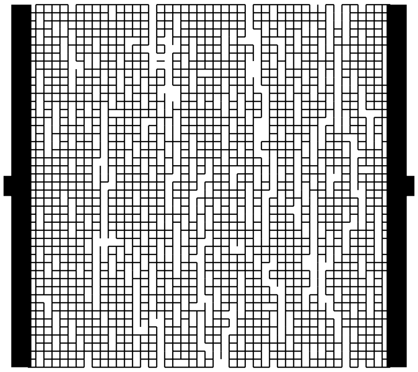

Fig. 1. Damage pattern with the film close to breakdown for a network with size 40 × 40 under a constant current of 1 A. The resistance is about 4 times the initial value. Broken bonds indicate the defects in the film.

For the present purposes, an excess-noise temperature T_N can be defined as:

$$T_N = \frac{S_V}{4 k_{\mathrm{B}} R}, \tag{5}$$

where $R = (1/I^2) \sum_\alpha r_\alpha i_\alpha^2$ is the total resistance of the network. Therefore, from equations (2) to (5), the excess-noise temperature can be written as:

$$T_N = T_0 + A R I^2\ \Phi_R, \tag{6}$$

$$\Phi_R = \frac{\sum_\alpha r_\alpha^2 i_\alpha^4}{\left(\sum_\alpha r_\alpha i_\alpha^2\right)^2} \tag{7}$$

where for the perfect network Φ_R takes the minimum value of $1/[N(N+1)]$. On the other hand, we can define an average temperature of the film as $T_{\mathrm{av}} = T_0 + \Delta T_{\mathrm{av}}$ where, according to equation (2), the average heating of the network ΔT_{av} can be expressed as:

$$\Delta T_{\mathrm{av}} = \frac{1}{N(N+1)} \sum_\alpha \delta T_\alpha = \theta R I^2 \tag{8}$$

where $\theta = A/[N(N+1)]$ represents the structure thermal resistance [25,26]. By introducing the increase of the noise temperature with respect to its equilibrium value, $\Delta T_N = T_N - T_0$ we can define a merit factor of the sample, F as:

$$F = \frac{\Delta T_N}{\Delta T_{\mathrm{av}}} \tag{9}$$

where $F \geq 1$. Therefore, values of the merit factor larger than unity are associated with an increasing dishomogeneity of the network.

As reasonable values of the parameters we take in these calculations: $N = 100$, $r_\alpha = 1\ \Omega$, $E_0 = 0.19\ eV$, $A = 5 \times 10^5\ K/W$, 77 K $\leq T_0 \leq$ 500 K and 0.5 A $\leq I \leq$ 2.0 A. The threshold of electrical breakdown has been fixed when the resistance increases over a factor of 10^3 with respect to the initial value. Further details about the simulations can be found in references [22,23].

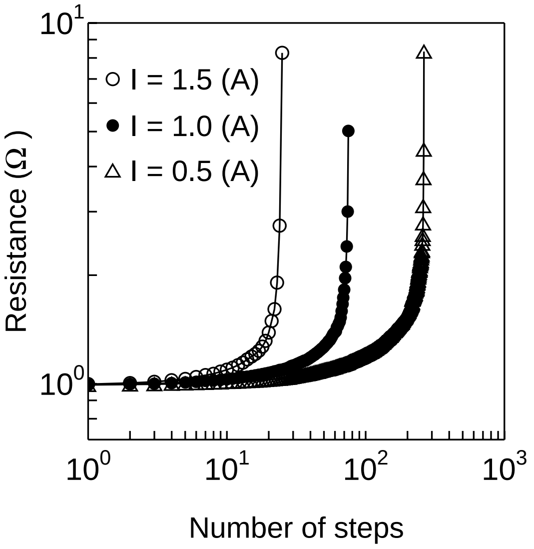

Fig. 2. Typical evolution of the network resistance for different biasing currents at $T_0 = 300$ K. Open circles, full circles and triangles refer respectively to $I = 1.5, 1.0, 0.5$ A.

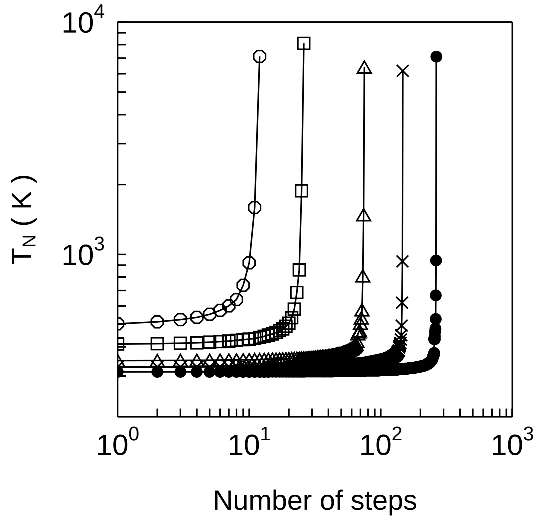

Fig. 3. Typical evolutions of the excess-noise temperature T_{N} for different biasing currents at $T_0 = 300$ K. Open circles, squares, triangles, crosses and full circles refer respectively to $I = 2.00, 1.50, 1.00, 0.75$ and 0.50 A.

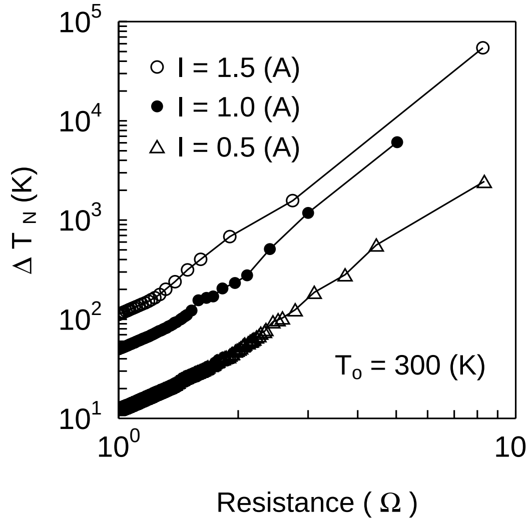

Fig. 4. Typical increase of ΔT_N as a function of the film resistance during the process of degradation at $T_0 = 300$ K. The three curves correspond respectively to $I = 1.5$ A (open circles), $I = 1.0$ A (full circles) and $I = 0.5$ A (triangles). For all the curves $R = 1$ Ω is the value corresponding to the perfect lattice

The results of the simulations are reported in Figures 1 to 6. Figure 1 shows a damaged pattern near breakdown of the network which exhibits the typical filamentary structure [19,22].

Figure 2 shows the degradation of the RRN resistance simulated at different values of the current flowing in the sample. By considering the number of iteration steps proportional to a given time scale (the units of which should be defined appropriately), the number of iteration steps needed to reach the breakdown, N_{Br}, can be identified with the lifetime of the RRN [23]. Therefore we can see that at increasing current values the lifetime is drastically reduced, while the breakdown is achieved in a similar way. This is related to the fact that we found the scaling exponent t in the power-law $R \propto (p - p_{\mathrm{c}})^t$ independent from the biasing current value even in the case of biased percolation, where, as usual, we indicate with p the fraction of broken resistors at an arbitrary step normalized to the total number of network resistors and with p_{c} the critical fraction value (percolation threshold) [17].

Figure 3 reports typical evolutions of the excess-noise temperature T_N. Again different values of the current flowing in the sample are considered ranging from 0.5 A to 2. A. In any case, in proximity of the electrical breakdown of the RRN, the excess-noise temperature exhibits a steep divergence.

Figure 4 illustrates an important feature of the degradation process: here ΔT_N is reported as a function of the corresponding RRN resistance. The three simulations refer to different values of the biasing current but for all $T_0 = 300$ K. The lowest value of the resistance corresponds to the perfect network. As shown in this figure, the excess-noise temperature can be expressed as a power-function of the sample resistance. Within the biased percolation model, independently of the value of I, we have found that $\Delta T_N \propto R^s$ where the scaling exponent s is 2.9 ± 0.4 for $T_0 = 300$ K, while for $T_0 = 77$ K we have found $s = 3.1 \pm 0.3$. The value larger than unity of the exponent s is associated with a higher sensitivity of the noise with respect to the resistance in monitoring the degradation process, a feature which is common with $1/f$ noise [5–17]. This property is physically justified by the fact that excess noise in both cases comes from higher moments of the current distribution with respect to the second

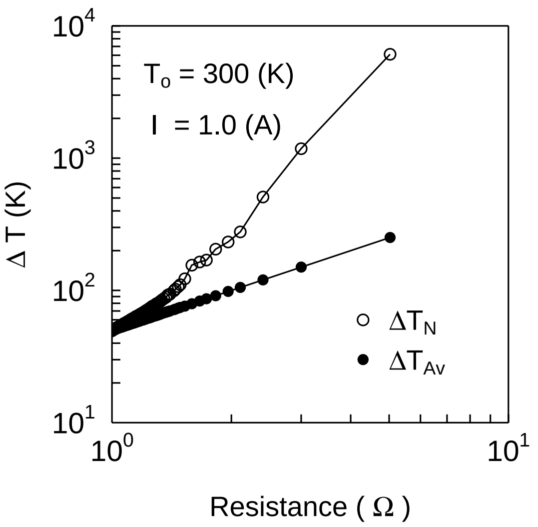

Fig. 5. Typical evolutions of ΔT_N (open circles) and of ΔT_{av} (full circles) as a function of the film resistance. Calculations refer to $I = 1.0$ A and $T_0 = 300$ K.

order moment determining the network resistance [13–18].

Figure 5 compares the typical evolutions of ΔT_N and of ΔT_{av} as a function of the increase of the sample resistance. At the beginning of the evolution, when the sample is substantially homogeneous, ΔT_N and ΔT_{av} practically coincide. Indeed, at this stage only few defects, uniformly distributed [22,23], are present in the film. As the degradation proceeds, the number of defects increases and their distribution departs significantly from being uniform. In fact, within the biased percolation model, the defects grow preferentially around already existing ones thus exhibiting a clustering attitude [22,23]. Therefore, as a result of the dishomogeneity of the sample and of the different type of local current average defining the noise and the average temperatures of the sample, the excess-noise temperature becomes systematically larger than the average temperature.

Figure 6 shows the final lifetime of 50 samples (different realizations of failure) as a function of the increase of the excess-noise temperature calculated at the 14-th step (intermediate step). The figure refers to a RNN biased by an external current $I = 1.5$ A and deposited on a substrate at temperature $T_0 = 300$ K. We can see that higher ΔT_N values at the intermediate step are associated with shorter final lifetimes. Similar behaviours are also found for other values of I and T_0 and even at relatively early iteration steps.

3 Conclusions

We have introduced a new type of excess noise related to non-homogeneous Joule heating of a RRN where each elemental resistance is associated with a local source of thermal noise. Numerical results are thus performed through a Monte Carlo simulations based on the biased percolation model. Due to non-homogeneous heating effects, for a given sample the excess-noise temperature is found to increase more than its average temperature, and substantially over the thermal temperature of the substrate. Such an increase of the excess-noise temperature exhibits a remarkable power-law with the resistance with a scaling exponent about 3. From an applicative point of view, the noise temperature so introduced can represent a suitable measure of the sample defectiveness. We remark that the specific correlation between the excess thermal-noise and the sample homogeneity makes the analysis of this quantity complementary to that of the standard $1/f$ noise where the physical interpretation of the noise source remains an unsolved problem in general [1,4]. Of course, for a wider application of this model one needs to better refine the physical description (*e.g.* by introducing the temperature dependence of the elemental resistors and their possible thermal interaction). Some of these issues will be the main topics of further research.

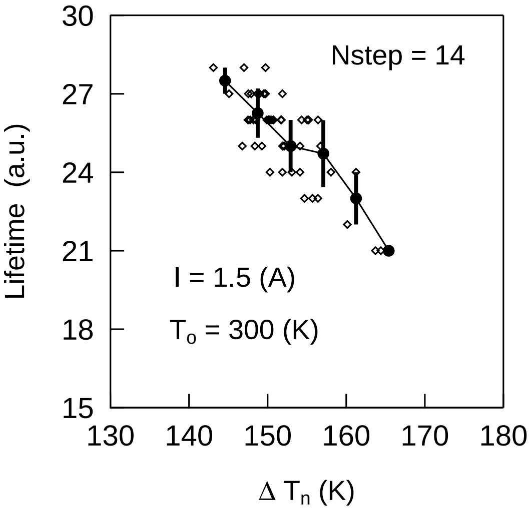

Fig. 6. Lifetimes of 50 samples as a function of the increase of the excess-noise temperature ΔT_N at the iteration step $N_{\text{step}} = 14$. The biasing current and the substrate temperature are respectively $I = 1.5$ A and $T_0 = 300$ K. Diamonds represent the lifetimes of the different samples, full circles their average values. The length of error bars is twice the standard deviation.

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